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Soft gluon resummation in transverse momentum space for electroweak boson production at hadron colliders

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Abstract. The distribution of W and Z bosons produced with small transverse momentum (p_{T}) at hadron colliders receives important contributions from large logarithms arising from soft gluon emission. Although conventionally the all-orders resummation of these 'Sudakov' logarithms is performed in impact parameter (Fourier transform) space, p_{T}-space resummation is also possible, and offers certain advantages. We present a detailed phenomenological analysis of W and Z production at small p_{T} at the Tevatron $p\bar{p}$ collider, using p_{T}-space resummation. A good description of the CDF and D0 data can be obtained provided a significant non-perturbative contribution is included. We also present predictions for the LHC.

1 Introduction

The description of gauge boson production at hadron colliders has recently attracted much theoretical interest, especially in the light of future high precision experiments at the Tevatron and the LHC [1–3]. Reliable predictions can only be obtained if soft gluon radiation effects are correctly taken into account. Theoretically the soft gluon emission manifests itself in the presence of large logarithmic corrections (Sudakov logarithms). For the particular case of the transverse momentum (p_{T}) distribution of a boson produced with invariant mass Q, the Sudakov logarithms are the logarithms of the ratio $Q^2/{p_{\mathrm{T}}}^2$. In the small p_{T} limit the logarithms diverge and the standard fixed-order perturbation theory approach breaks down. However, a finite result can be recovered if the soft gluon emission is accounted for to all orders in α_s. This is achieved by resumming the logarithmic corrections.

Resummation can be performed either directly in transverse momentum (p_{T}) space or in the Fourier conjugate impact parameter (b) space. The most leading logarithmic contributions, of the form $\alpha_s^n \ln^{2n-1}(Q^2/{p_{\mathrm{T}}}^2)$, can be directly resummed in p_{T} space (the so-called Double Leading Logarithm Approximation) [4]. The impact parameter (b) space method [5] allows one to resum subleading logarithms, including those 'kinematic' logarithms arising as a direct result of transverse momentum conservation [6]. Although very successful theoretically, the b space method suffers from certain deficiencies and drawbacks which need to be 'fixed' in order to obtain a satisfactory agreement between the theoretical predictions and experimental data. For example, one experiences difficulties when matching the resummed (small p_{T}) and fixed-order (large p_{T}) predictions. Moreover, it is impossible to make predictions for *any* value of p_{T} without a prescription dealing with the non-perturbative regime of large b[1]. These difficulties can be naturally circumvented if the resummation is performed in p_{T} space [8]. Unfortunately the p_{T} space methods which have been developed so far for resumming subleading logarithms are derived from the b space approach [8–10], and as such they simply provide an approximation to the b space result. However, the goal is to develop a phenomenologically useful p_{T} space expression that reproduces *all* the good features of the b space resummation without the drawbacks related to this method.

In a previous paper [10] we proposed a p_{T} space resummation formalism at the parton level which resums the first four 'towers' of logarithms (i.e. terms of the form $\alpha_s^n \ln^{2n-m}(Q^2/{p_{\mathrm{T}}}^2)$, $m = 1,..,4$), *including* the effects of transverse momentum conservation. The differences between our formalism (KS) and other p_{T} space approaches (FNR [9] and EV [8]) were discussed in [11]. Here we want to concentrate on the practical applications of our formalism, in particular on the comparison with available Tevatron data.

In our analysis we use the most recent sets of CDF [12] and D0 data [13] on Z production and D0 data [14] on W production. Since we are only interested here in the resummed part of the cross section and do not perform matching with the fixed-order part, we do not consider

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[1] Recently it has been argued [7] that the b space drawbacks discussed here are less relevant at Tevatron and LHC energies

data above $p_{\rm T} > 25$ GeV. In this $p_{\rm T}$ range the resummed part accounts for almost the entire cross section.

In a manner similar to the b space formalism, the $p_{\rm T}$ space formalism is incomplete without a prescription for dealing with the non-perturbative effects. Indeed previous phenomenological analyses have shown that the very small $p_{\rm T}$ region is dominated by non-perturbative contributions. Here we use the method of introducing non-perturbative effects in $p_{\rm T}$ space first proposed in [8]. We investigate the form and size of the non-perturbative contributions obtained from fits to the data. We finish our investigations by commenting on W and Z boson production at the LHC.

2 Theoretical cross section for $p\bar{p}\to W, Z + X$

In this section we summarise the derivation of the main theoretical results for $p_{\rm T}$ space used in fits to the data. For the discussion of the b space results the reader is referred to [5,15].

The resummed part of the theoretical cross section in $p_{\rm T}$ space for a Drell-Yan-type process follows from the b space formula, cf. [5]

$$\frac{d\sigma}{dp_{\rm T}{}^2\,dQ^2} = \frac{\sigma_0}{Q^2}\sum_q e_q^2 \int_0^1 dx_A\,dx_B\,\delta\left(x_A x_B - \frac{Q^2}{s}\right) \times\frac{1}{2}\int_0^\infty db\,b\,J_0(p_{\rm T}b)\,\exp[\mathcal{S}(b,Q)] \times\tilde{f}'_{q/A}\left(x_A,\frac{b_0}{b}\right)\,\tilde{f}'_{\bar{q}/B}\left(x_B,\frac{b_0}{b}\right)\,. \quad (1)$$

with $\sigma_0 = 4\pi\alpha^2/(9s)$, $b_0 = 2\exp(-\gamma_E)$, and where

$$S(b,Q^2) = -\int_{\frac{b_0^2}{b^2}}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A(\alpha_S(\bar{\mu}^2)) + B(\alpha_S(\bar{\mu}^2))\right], \quad (2)$$

$$A(\alpha_S) = \sum_{i=1}^\infty\left(\frac{\alpha_S}{2\pi}\right)^i A^{(i)}\,,$$

$$B(\alpha_S) = \sum_{i=1}^\infty\left(\frac{\alpha_S}{2\pi}\right)^i B^{(i)}\,. \quad (3)$$

We first consider the non-singlet (NS) cross-section, i.e. we introduce

$$\tilde{f}'_{q/H} = f'_{q/H} - f'_{\bar{q}/H}$$

as modified higher-order NS parton distributions. The modified parton distributions are related to the $\overline{\rm MS}$ parton distributions, f, by a convolution [5,16,15]

$$f'_{a/H}(x_A,\mu) = \sum_c\int_{x_A}^1\frac{dz}{z}\,C_{ac}\left(\frac{x_A}{z},\mu\right)f_{c/H}\,(z,\mu)\;, \quad (4)$$

where $(a,b\neq g)$

$$C_{ab}(z,\mu) = \delta_{ab}\left\{\delta(1-z) + \bar{\alpha_s}(\mu)C_F \times\left[1-z+\left(\frac{\pi^2}{2}-4\right)\delta(1-z)\right]\right\},$$

$$C_{ag}(z,\mu) = \bar{\alpha_s}(\mu)T_R\Big[2z(1-z)\Big],$$

and $\bar{\alpha_s}(\mu) = \frac{\alpha_s(\mu)}{2\pi}$, $C_F = 4/3$, $T_R = 1/2$.

The N-th moment of the cross section with respect to $\tau = Q^2/s$ has the form

$$\mathcal{M}(N) = \int d\tau\,\tau^N\frac{Q^2}{\sigma_0}\frac{d\sigma}{dp_{\rm T}{}^2\,dQ^2} = \sum_q e_q^2\frac{1}{2}\int_0^\infty db\,b\,J_0(p_{\rm T}b) \times\exp\left[\mathcal{S}(b,Q)\right]\tilde{f}'_{q/A}\left(N,\frac{b_0}{b}\right)\,\tilde{f}'_{\bar{q}/B}\left(N,\frac{b_0}{b}\right). \quad (5)$$

Solving the DGLAP equation for the N-th moment of the modified parton distribution $\tilde{f}'_{q/H}(N,Q) = \int_0^1 dx_H x_H^N\,\tilde{f}'_{q/H}(x_H,Q)$, and integration by parts lead to (cf. [8])

$$\mathcal{M}(N) = \frac{d}{dp_{\rm T}{}^2}\left\{\sum_q e_q^2\,\tilde{f}'_{q/A}(N,p_{\rm T})\tilde{f}'_{\bar{q}/B}(N,p_{\rm T}) \times\int_0^\infty dx\,J_1(x) \times\exp\left[\mathcal{S}(x,Q) - 2\int_{\frac{b_0^2 p_{\rm T}{}^2}{x^2}}^{p_{\rm T}{}^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma'_N(\bar{\alpha_s}(\bar{\mu}))\right]\right\},$$

where $x = p_{\rm T}b$.

In order to obtain an expression for the hadron level cross section, the following approximation is introduced

$$\exp\left[\mathcal{S}(x,Q) - 2\int_{\frac{b_0^2 p_{\rm T}{}^2}{x^2}}^{p_{\rm T}{}^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma'_N(\bar{\alpha_s}(\bar{\mu}))\right] \approx \exp\left[\mathcal{S}(x,Q)\right]\,. \quad (6)$$

The above equality is exact for the first four towers of logarithms; it is only the fifth tower that contains the first modified anomalous dimension coefficient $\gamma'^{(1)}_N$. This can be easily seen by expanding the exponential in (6) (assuming here a fixed coupling constant for simplicity)

$$\exp\left[\mathcal{S}(x,Q) - 2\int_{\frac{b_0^2 p_{\rm T}{}^2}{x^2}}^{p_{\rm T}{}^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma'_N(\bar{\alpha_s}(\bar{\mu}))\right] = \sum_{N=0}^\infty\frac{(-1)^N}{N!}\left[\frac{1}{2}(A^{(1)}\bar{\alpha_s} + A^{(2)}\bar{\alpha_s}^2 + ...)(L+L_b)^2 + (B^{(1)}\bar{\alpha_s} + B^{(2)}\bar{\alpha_s}^2 + ...)(L+L_b) + 2(\gamma'^{(1)}_N\bar{\alpha_s} + \gamma'^{(2)}_N\bar{\alpha_s}^2 + ...)L_b\right]^N,$$

where $L = \ln(Q^2/{p_\mathrm{T}}^2)$ and $L_b = \ln(x^2/b_0^2)$. The first term containing $\gamma_N'^{(1)}$ which does not vanish after integration over x is of the form $\bar{\alpha_s}^N A^{(1)^{N-1}} \gamma_N'^{(1)} L^{2(N-2)} L_b^3$. The same statement holds also for the singlet parton distribution functions.

The resulting expression

$$\mathcal{M}(N) = \frac{d}{d{p_\mathrm{T}}^2}\left\{ \sum_q e_q^2\, \tilde{f}'_{q/A}(N,p_\mathrm{T})\tilde{f}'_{\bar{q}/B}(N,p_\mathrm{T}) \times \int_0^\infty dx\, J_1(x)\, \exp[\mathcal{S}(x,Q)] \right\} \tag{7}$$

can now be transformed back to momentum space by the means of the inverse Mellin transform

$$\frac{d\sigma}{d{p_\mathrm{T}}^2\, dQ^2} = \frac{\sigma_0}{Q^2}\sum_q e_q^2 \int_0^1 dx_A\, dx_B\, \delta\left(x_A x_B - \frac{Q^2}{s}\right) \times \frac{d}{d{p_\mathrm{T}}^2}\left\{ \int_0^\infty dx\, J_1(x)\, \exp[\mathcal{S}(x,Q)] \times \tilde{f}'_{q/A}(x_A,p_\mathrm{T})\, \tilde{f}'_{\bar{q}/B}(x_B,p_\mathrm{T}) \right\}. \tag{8}$$

At the parton level we calculated the quantity[2]

$$\begin{aligned}
\frac{1}{\sigma_0}\frac{d\sigma}{d{p_\mathrm{T}}^2} &= -\frac{1}{2{p_\mathrm{T}}^2}\int_0^\infty dx\, x\, J_1(x)\frac{d}{dx}\exp[\mathcal{S}(x,Q)] \\
&= -\frac{1}{2{p_\mathrm{T}}^2}\exp(\mathcal{S}_\eta(Q)) \times \int_0^\infty dx\, x\, J_1(x)\frac{d}{dx}\exp[\tilde{\mathcal{S}}(x,Q)] \\
&= \frac{\alpha_s(\mu^2)A^{(1)}}{2{p_\mathrm{T}}^2\pi} e^{\mathcal{S}_\eta} \sum_{N=1}^\infty \left(\frac{-\alpha_s(\mu^2)A^{(1)}}{\pi}\right)^{N-1} \\
&\quad \times \sum_{m=0}^{N-1}\frac{1}{m!}\sum_{k=0}^{N-m-1}\frac{1}{k!}\sum_{l=0}^{N-m-k-1}\frac{1}{l!} \\
&\quad \times \sum_{j=0}^{N-m-k-l-1}\frac{1}{j!}\sum_{i=0}^{N-m-k-l-j-1} \\
&\quad \times \frac{1}{i!(N-m-k-l-j-i-1)!} \\
&\quad \times c_2^m c_3^k c_4^l c_5^j c_6^i c_1^{N-m-k-l-j-i-1} \\
&\quad \times \sum_{n=1}^6 n c_n \tau_{N+m+2k+3l+4j+5i+n-2} \\
&\equiv -\frac{1}{2{p_\mathrm{T}}^2}\Sigma_1(p_\mathrm{T},Q)\,,
\end{aligned} \tag{9}$$

in terms of resummed towers of logarithms in p_T space. Here $\tilde{\mathcal{S}}(x,Q) = \mathcal{S}(x,Q) - \mathcal{S}_\eta(Q)$ with $\mathcal{S}_\eta$ and c coefficients defined in [10]. The expression for $\int_0^\infty dx J_1(x)\exp[\mathcal{S}(x,Q)]$ can be derived in a similar manner

$$\begin{aligned}
&\int_0^\infty dx J_1(x)\exp[\mathcal{S}(x,Q)] \\
&= \exp(\mathcal{S}_\eta)\sum_{N=1}^\infty \left(\frac{-\alpha_S(\mu^2)A^{(1)}}{\pi}\right)^{N-1} \sum_{m=0}^{N-1}\frac{1}{m!}\sum_{k=0}^{N-m-1}\frac{1}{k!} \\
&\quad \times \sum_{l=0}^{N-m-k-1}\frac{1}{l!}\sum_{j=0}^{N-m-k-l-1}\frac{1}{j!}\sum_{i=0}^{N-m-k-l-j-1} \\
&\quad \times \frac{1}{i!(N-m-k-l-j-i-1)!} \\
&\quad \times c_2^m c_3^k c_4^l c_5^j c_6^i c_1^{N-m-k-l-j-i-1}\tau_{N+m+2k+3l+4j+5i-1} \\
&\equiv \Sigma_2(p_\mathrm{T},Q)\,,
\end{aligned} \tag{10}$$

Finally we arrive at the p_T space formula for the Drell-Yan cross section at the hadron level

$$\frac{d\sigma}{d{p_\mathrm{T}}^2\, dQ^2} = \frac{\sigma_0}{Q^2}\sum_q e_q^2\int_0^1 dx_A\, dx_B\, \delta\left(x_A x_B - \frac{Q^2}{s}\right) \times \frac{d}{d{p_\mathrm{T}}^2}\left\{\Sigma_2(p_\mathrm{T},Q)\, f'_{q/A}(x_A,p_\mathrm{T})\, f'_{\bar{q}/B}(x_B,p_\mathrm{T})\right\}. \tag{11}$$

In what follows we will refer to the result in (11) as the KS hadron-level formula in p_T space.

In principle, the parton level formula (9) allows us to resum *any* number of towers of logarithms. In practice, however, the fifth tower of logarithms cannot be fully taken into account due to the lack of knowledge of the coefficient $A^{(3)}$. Since our approximation (6) is valid only up to the fifth tower too, (11) can be used to resum the first four towers of logarithms, in other words the summation in (10) stops at $N = 4$. In [10], the contributions from fifth and higher towers were estimated to be numerically very small in the region of p_T of interest.

The analogous expression for the transverse momentum distribution of a massive vector boson V produced in $p\bar{p}\to V + X$ is

$$\frac{d\sigma}{dp_\mathrm{T}} = \sigma_0\sum_{qq'} U^V_{qq'}\int_0^1 dx_A\, dx_B\, \delta\left(x_A x_B - \frac{M_V^2}{s}\right) \times \frac{d}{dp_\mathrm{T}}\left\{\Sigma_2(p_\mathrm{T},M_V)\, f'_{q/A}(x_A,p_\mathrm{T})\, f'_{q'/B}(x_B,p_\mathrm{T})\right\}, \tag{12}$$

where

$$\sigma_0 = \frac{\pi\sqrt{2}G_F}{N}$$
$$U^V_{qq'} = \begin{cases} |V_{qq'}|^2 & V = W^\pm\,, \\ (V_q^2 + A_q^2)\delta_{qq'} & V = Z\,, \end{cases} \tag{13}$$

where $V_{qq'}$ denotes the appropriate CKM matrix element, and V_q, A_q are the vector and axial couplings of the Z boson to quarks.

[2] see (21) in [10]

In practice it is convenient to split the differentiation in (12) into two terms

$$\frac{d\sigma}{dp_{\mathrm{T}}} = \sigma_0 \sum_{qq'} U^V_{qq'} \int_0^1 dx_A\, dx_B\, \delta\left(x_A x_B - \frac{M_V^2}{s}\right) \times \left\{ -\frac{1}{p_{\mathrm{T}}} \Sigma_1(p_{\mathrm{T}}, M_V) f'_{q/A}(x_A, p_{\mathrm{T}})\, f'_{q'/B}(x_B, p_{\mathrm{T}}) + \Sigma_2(p_{\mathrm{T}}, M_V) \frac{d}{dp_{\mathrm{T}}} \left[f'_{q/A}(x_A, p_{\mathrm{T}})\, f'_{q'/B}(x_B, p_{\mathrm{T}}) \right] \right\}. \tag{14}$$

This trick allows us to apply an inevitable numerical derivative only to the product of the parton distributions and not to the whole expression, leading to a reduction of the numerical error.

2.1 Inclusion of the non-perturbative effects in p_{T} space

The form of the non-perturbative ansatz in p_{T} space is expected to be important only in regions where perturbation theory fails, i.e. at the very low values of $p_{\mathrm{T}} \leq 2-3$ GeV. In contrast, the higher p_{T} region can be described purely by the resummed perturbative QCD expression. We choose to incorporate the low energy effects using the form of the p_{T} space non-perturbative function $\tilde{F}^{NP}(p_{\mathrm{T}})$ advocated in [8]

$$\tilde{F}^{NP}(p_{\mathrm{T}}) = 1 - \exp\left[-\tilde{a}\, {p_{\mathrm{T}}}^2\right]. \tag{15}$$

The role of this function is to account for the distribution in the very low p_{T} region, and here we are assuming that the shape is approximately gaussian. However in order to combine this with the perturbative result, the latter needs to be 'frozen' or 'switched off' at some critical value of p_{T} where the coupling α_s becomes large. A similar freezing is required in the b space approach where the coupling is effectively $\alpha_s(1/b)$. In other words we require not only (i) a form $\tilde{F}^{NP}(p_{\mathrm{T}})$ for the distribution in the non-perturbative region, but also (ii) a prescription for moving smoothly from the perturbative to the non-perturbative region. One possibility for the latter is the 'freezing' prescription of [8]

$$p_{\mathrm{T}*} = \sqrt{{p_{\mathrm{T}}}^2 + {p_{\mathrm{T\,lim}}}^2 \exp\left[-\frac{{p_{\mathrm{T}}}^2}{{p_{\mathrm{T\,lim}}}^2}\right]} \tag{16}$$

which has the property

$$p_{\mathrm{T}*} = \begin{cases} p_{\mathrm{T}}\,, & p_{\mathrm{T}} \gg p_{\mathrm{T\,lim}}\,, \\ p_{\mathrm{T\,lim}}\,, & p_{\mathrm{T}} \ll p_{\mathrm{T\,lim}}\,. \end{cases} \tag{17}$$

It is important to note that there are *two* pieces of information contained in this definition: the value of the limiting value $p_{\mathrm{T\,lim}}$ and the abruptness of the transition to this value. The use of a gaussian function in the definition (16), compared to say a power law function, implies a rapid transition that, as we shall see below, is consistent with the data.

Applying the above prescription to our expression (12) leads to

$$\frac{d\sigma}{dp_{\mathrm{T}}} = \sigma_0 \sum_{qq'} U^V_{qq'} \int_0^1 dx_A\, dx_B\, \delta\left(x_A x_B - \frac{M_V^2}{s}\right) \times \left\{ -\frac{1}{p_{\mathrm{T}*}} \frac{dp_{\mathrm{T}*}}{dp_{\mathrm{T}}} \Sigma_1(p_{\mathrm{T}*}, M_V) f'_{q/A}(x_A, p_{\mathrm{T}*}) \times f'_{q'/B}(x_B, p_{\mathrm{T}*}) \tilde{F}^{NP}(p_{\mathrm{T}}) + \Sigma_2(p_{\mathrm{T}*}, M_V) \frac{dp_{\mathrm{T}*}}{dp_{\mathrm{T}}} \times \frac{d}{dp_{\mathrm{T}*}} \left[f'_{q/A}(x_A, p_{\mathrm{T}*})\, f'_{q'/B}(x_B, p_{\mathrm{T}*}) \right] \tilde{F}^{NP}(p_{\mathrm{T}}) + \Sigma_2(p_{\mathrm{T}*}, M_V) f'_{q/A}(x_A, p_{\mathrm{T}*})\, f'_{q'/B}(x_B, p_{\mathrm{T}*}) \times \frac{d}{dp_{\mathrm{T}}} \tilde{F}^{NP}(p_{\mathrm{T}}) \right\}. \tag{18}$$

Note that the simple form of $\tilde{F}^{NP}(p_{\mathrm{T}})$ in the present framework does not take into account a possible dependence on Q and x. This is in contrast to the b space treatment of [21], where an x–dependent linear term in b was added to the argument of the gaussian non-perturbative function. It has also been argued [5] that the width of the non-perturbative gaussian distribution, in our case the parameter $1/\tilde{a}$, should increase linearly with $\log Q$[3]. Since in the present case we are only interested in W, Z production at a single collider energy, the values of Q and x are essentially fixed at M_V and $M_V/\sqrt{s}$ respectively. Therefore we are not able to say anything about the form of the dependence of the non-perturbative parameters on Q and x. Nor will we investigate different functional forms for $\tilde{F}^{NP}(p_{\mathrm{T}})$ — the simple gaussian form in (15) will allow perfectly good fits to the Tevatron data.

The lack of information on the x dependence of the non-perturbative contributions should be borne in mind when considering the predictions for W and Z production at the LHC.

3 Results and discussion

For the parton level cross section we have advocated [10] the use of the renormalization scale $\mu_R = {p_{\mathrm{T}}}^{(2/3)} Q^{(1/3)}$ as a means of eliminating certain logarithmic terms from the Sudakov factor and thus increasing the reliability of our approach. Since the renormalization scale determines the strength of the coupling in the theoretical predictions, it must somehow depend on the size of the transverse momentum and we require the choice of the scale to reflect this fact. Moreover, for values of p_{T} where perturbative QCD can be safely applied and for the values of Q considered here, such a μ_R is always bigger then the b quark

[3] Indeed, fits of a non-perturbative gaussian distribution to the low energy data [17] typically give much larger values of $\tilde{a}$ (≈ 0.55 GeV^{-2}), which suggests a strong dependence of $\tilde{a}$ on Q

mass, thus lessening the relevance of the correction due to the treatment of quark mass thresholds. Another obvious choice for the renormalization scale is $\mu_R = {p_\mathrm{T}}^4$. However, since we find only a very small dependence of the resummed part of the cross section on the choice of μ_R, from now on we use $\mu_R = {p_\mathrm{T}}^{(2/3)} Q^{(1/3)}$ as the default choice for the KS approach.

The Drell-Yan cross section (11) has been derived in the limit of a fixed number of quark flavours, N_f, which implies that no quark mass threshold effects are considered. In the original b space approach, the dependence on N_f enters in the Sudakov factor through the $A^{(2)}$, $B^{(2)}$ coefficients and through the β function in the expansion of α_s. For our p_T space method we propose to change N_f according to the number of flavours active at the renormalization scale at which α_s is calculated while the remainder of the expression is derived in the massless quark limit. With the choices of the scale we use this roughly corresponds to the energy scale of the emitted gluons and fits comfortably into the physical picture of the process. Changing the number of active quark flavours N_f as p_T varies immediately leads to the problem of obtaining reliable predictions free of unphysical discontinuities. To overcome this we use an analytically extended $\alpha_s^{\overline{\mathrm{MS}}}$ scheme which incorporates finite-mass quark threshold effects into the running of the coupling, as proposed by Brodsky et al. [19]. By connecting the coupling directly to the analytic and physically-defined effective charge scheme, the authors of [19] obtain an analytic expression for the effective number of flavours which is a continuous function of the renormalization scale and the quark masses.

The results presented below are for the Tevatron experiments, CDF and D0, at $\sqrt{s} = 1.8$ TeV. Unless stated otherwise we use the factorization scale $\mu_f = p_\mathrm{T}$, MRST99 parton distribution functions [20] (central gluon), branching ratios $BR(Z \to e^- e^+) = 3.366\%$, $BR(W \to e\nu) = 11.1\%$ and the world average value of the strong coupling $\alpha_s(M_Z) = 0.1175$. To normalize the theory predictions to the data we take only those experimental points with $p_\mathrm{T} < 15$ GeV.

There is a significant amount of Tevatron data on W and Z production that should, in principle, allow a precise determination of the non-perturbative parameters from fits to the data. However since the measurement of the W transverse momentum requires correcting for detector effects that are much stronger that in the Z measurement case, for the purpose of this analysis we take only the Z data. Again, we consider only those experimental points with $p_\mathrm{T} < 15$ GeV for the fit range. The overall normalization is taken as a free parameter, since we are primarily concerned with the *shape* of the distributions[5].

Generally we find that the best χ^2/d.o.f. value is obtained by values of $\sqrt{1/\tilde{a}}$ and $p_{\mathrm{T\,lim}}$ of order $3-4$ GeV. In this context the values proposed by the EV collaboration $\tilde{a} = 0.1$ GeV^{-2}, $p_{\mathrm{T\,lim}} = 4$ GeV provide one of the best fits

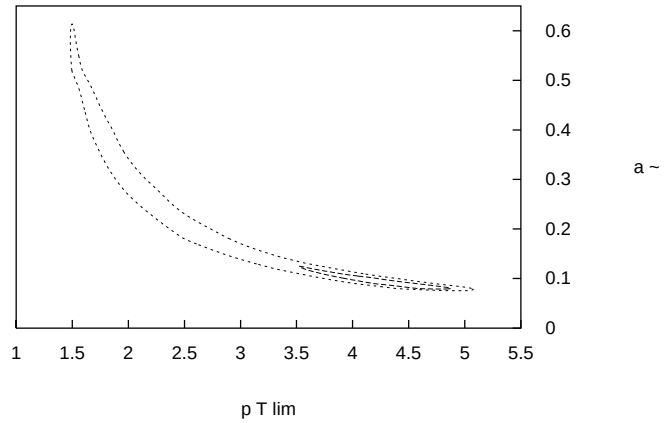

Fig. 1. The contours of equal χ^2 in the $\tilde{a}$, $p_{\mathrm{T\,lim}}$ plane for the KS p_T space approach with the non-perturbative input of the form (15,16). Both CDF and D0 data (with separate normalization) for $p_\mathrm{T} < 15$ GeV are used in the fit. The outer and inner contours correspond to χ^2/d.o.f. = 1 and 0.75 respectively

and describe the Z data well. This is also in agreement with the CDF and D0 analysis, cf. [12,13]. Furthermore we find that there is a wide range of strongly correlated values of larger $\tilde{a}$ and smaller $p_{\mathrm{T\,lim}}$ for which χ^2/d.o.f. is only minimally worse. This is illustrated in Fig. 1, which shows the equal χ^2 contours in the plane for χ^2/d.o.f. = 1 and 0.75.

The correlation between $\tilde{a}$ and $p_{\mathrm{T\,lim}}$ is easy to understand. Increasing $p_{\mathrm{T\,lim}}$ corresponds to requiring the non-perturbative contribution to describe the data out to a larger value of p_T, and therefore a broader gaussian distribution (equivalently, smaller $\tilde{a}$) is required. The fact that the fit deteriorates sharply as $p_{\mathrm{T\,lim}}$ is made very small shows that ('frozen') perturbation theory alone cannot describe the data over the whole p_T range.

Given the large variation in the allowed values of $\tilde{a}$ and $p_{\mathrm{T\,lim}}$, it is difficult to gauge the predictive power of these results, especially when one allows for a possible additional x and Q dependence. We also find that with the current experimental data there is no need to introduce additional overall smearing, as proposed in [8]. A modification of $\tilde{F}^{NP}(p_\mathrm{T})$, such as adding a linear term in the exponential or using a different freezing method, does not significantly improve the fit either.

In Figs. 2, 3, 4 we present a comparison between experimental data on Z production as measured by CDF and D0, W production as measured by D0, and various theoretical distributions calculated using (i) the b space method, (ii) the EV p_T space method, and (iii) the KS p_T space method. We observe good agreement between the data and the theoretical predictions for all three methods, in the range of $p_\mathrm{T} = 0 \sim 25$ GeV. In general, the b space distribution is more 'peaked' than the p_T space equivalents. This effect is, however, very susceptible to the choice of the non-perturbative function and values of the non-perturbative parameters. The b space distribution is also higher in the intermediate range of $p_\mathrm{T} = 10 \sim 20$

[4] Other choices of μ_R have been considered in the literature, for example the authors of [18] proposed to take $\mu_R = \sqrt{{p_\mathrm{T}}^2 + Q^2}$

[5] The *total* W and Z cross sections are known to be well described by NNLO perturbation theory, see e.g. [20]

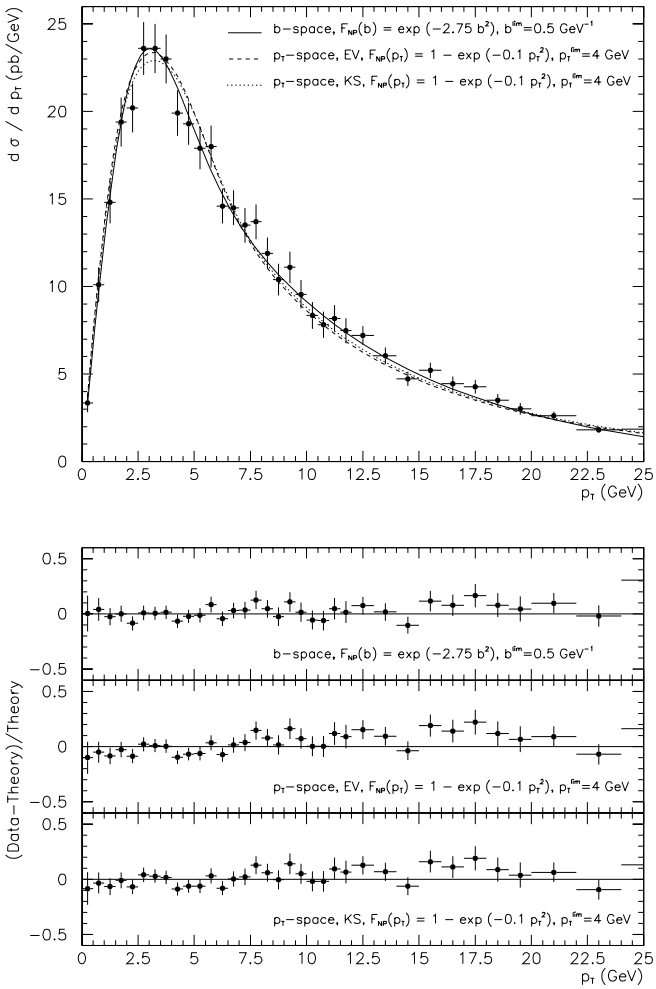

Fig. 2. Comparison between CDF data on Z production and theoretical predictions for the b space method, p_{T} space method in the EV approach and in the KS approach. For the b space method we use an effective gaussian form of F^{NP} as in [15]

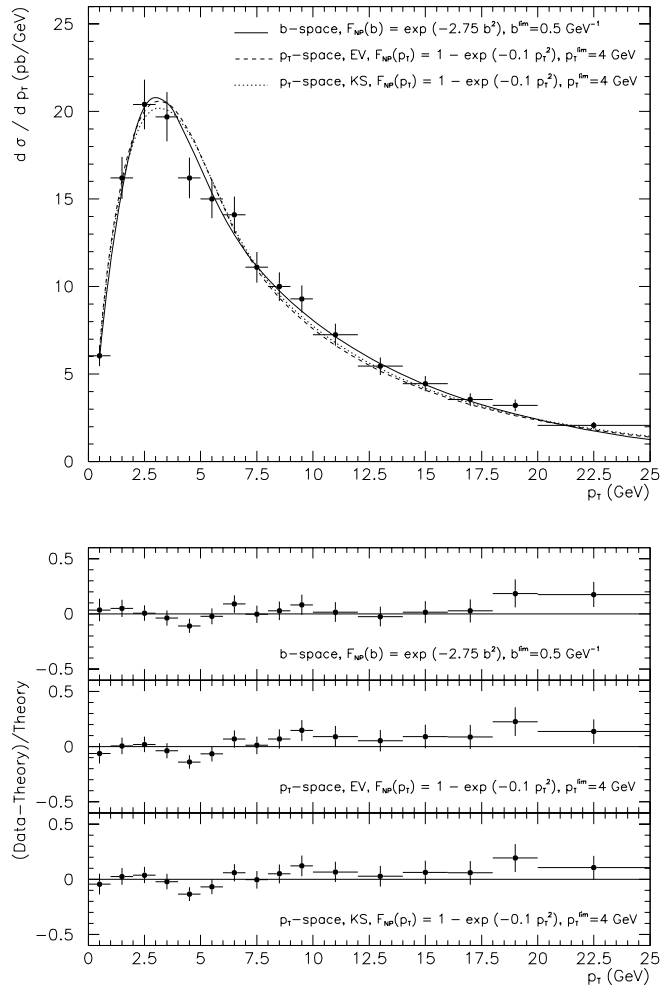

Fig. 3. Comparison between D0 data on Z production and theoretical predictions for the b space method, p_{T} space method in the EV approach and in the KS approach. For the b space method we use an effective gaussian form of F^{NP} as in [15]

GeV, where the non-perturbative physics does not influence the resummed perturbative result. In this region the KS distribution approximates the b space result better than the corresponding EV distribution. Given that the KS formalism resums more towers of logarithms than the EV formalism, this is an expected result. The increase of the cross section due to incorporating the fourth, NNNL, tower can be as big as 4% for some values of p_{T}, both for W and Z production [10]. Interestingly we also observe a significant sensitivity to the value of $\alpha_s(M_Z)$ used in the calculations. A variation of $\alpha_s(M_Z)$ by ± 0.005 around its average value, 0.1175, can cause, for some values of p_{T}, a more than $\pm 8\%$ change in the Z p_{T} distribution.

The transverse momentum distribution of W's and Z's at the LHC, predicted in the p_{T} space formalism (KS), is shown in Fig. 5. The results agree with similar analyses performed using the b space method [2]. For the sake of this analysis we used the standard Tevatron values of the non-perturbative parameters $\tilde{a} = 0.1$ GeV^{-2} and $p_{\mathrm{T\,lim}} = 4$ GeV in p_{T} space. This may prove to be a very unwise assumption if the non-perturbative parameterization does depend significantly on the partons momentum fractions, for example in the way it was proposed for the b space method ([21]). However it does provide a useful benchmark and a reasonable 'first guess'.

4 Summary

We have applied the KS resummation technique in p_{T} space, developed in [10], to the hadronic production of vector bosons. At the hadron level our approach retains the potential of the full resummation of the first four towers of logarithms. We also allow for a non-perturbative contribution, with a smooth interpolation between the perturbative and non-perturbative regimes at small p_{T}. Our numerical results generally show good agreement with recent data on W and Z boson production from the Tevatron collider.

For the resummed part of the $d\sigma/dp_{\mathrm{T}}$ distribution we observe rather weak dependence on the renormalization

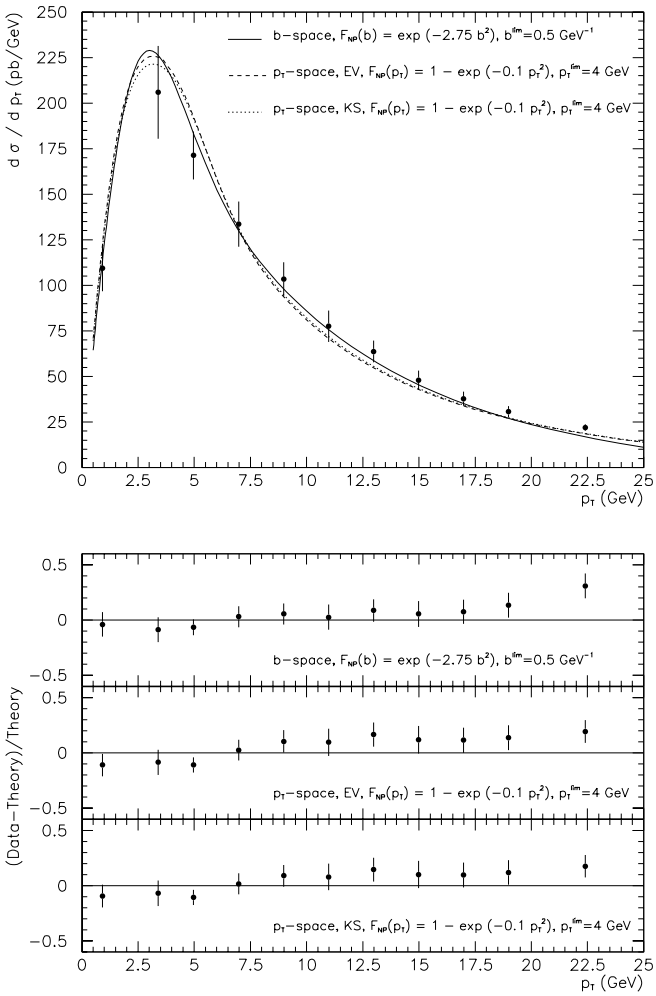

Fig. 4. Comparison between D0 data on W production and theoretical predictions for the b space method, p_{T} space method in the EV approach and in the KS approach. For the b space method we use an effective gaussian form of F^{NP} as in [15]

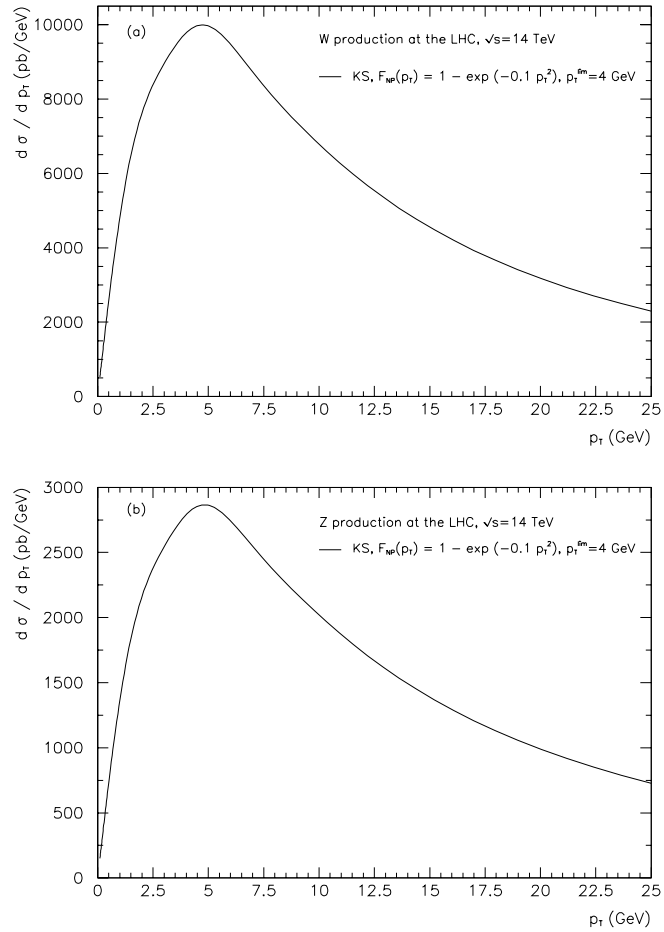

Fig. 5a,b. p_{T} space predictions (KS formalism) for the transverse momentum distribution at the LHC of: **a** W boson, **b** Z boson

scale, but some sensitivity to the value of $\alpha_s(M_Z)$. However the non-perturbative contribution, which at present must be determined from fits to data, is not well determined. We have not attempted to estimate the full error on the non-perturbative contribution. Rather, we showed that a simple Gaussian form, with reasonable values of the parameters, gives an acceptable fit. The width of the gaussian and the transition point between the perturbative and non-perturbative regions are, however, strongly correlated. This, combined with the lack of knowledge of the x and Q dependence of the non-perturbative parameters, makes it difficult to formulate precise predictions for the corresponding distributions at the LHC.

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